Intermediate Social Statistics

Homework Assignment Answers for Week 5

Question One

What is the stochastic component of this model?

$$Prob(y_i = 1|\mathbf{x_i}) = \frac{e^{\pi}}{1+e^{\pi}} = \Lambda(\pi)$$

What is the systematic component of the model?

$$\pi = (\mathbf{x_i}\beta)$$

What is the difference in the expected probability of ruling in the governors' favor between two judges who are both from the governor's party but one of whom is elected and the other appointed?

$$Prob(y_i = 1|X_1 = \text{elected}; X_2 = \text{governor's party}) = \frac{1}{1+e^{-(.01+.07(1)+1.61(1)+.21(1)=.870)}}$$

$$Prob(y_i = 1|X_1 = \text{unelected}; X_2 = \text{governor's party} = \frac{1}{1+e^{-(.01+.07(0)+1.61(1)+.21(0)=.835)}}$$

The difference is $.870 - .835 = .035$

What is the difference in the expected probability of ruling in the governor's favor between two judges, both of whom are elected, but one who is from the governor's party and one who is not?

$$Prob(y_i = 1|X_1 = \text{elected};X_2 = \text{governor's party} = \frac{1}{1+e^{-(.01+.07(1)+1.61(1)+.21(1))}} = .870$$

$$Prob(y_i = 1|X_1 = \text{unelected};X_2 = \text{governor's party} = \frac{1}{1+e^{-(.01+.07(1)+1.61(0)+.21(0))}} = .520$$

The difference is $.870 - .520 = .35$

Question Two

We need to write down a function that describes how the data for an individual were generated - that describes the data generating process. Given that this is a series of zeros and ones, it probably makes sense to treat this as a series of Bernoulli trials with success rate p; or a binomial distribution modeling the aggregate outcome with success rate p and number of trials equal to n. We have to assume that each of the committee votes cast is an independent event. The variable is a series of Bernoulli trials: $1 =$ vote for a proposal; $0 =$ vote against proposal.

Let P=Pr(Y=1). From the given sample we want to estimate p, the probability of a vote for a proposal. Note that the sample mean is simply the proportion of votes for a proposal and can be expressed as....

$$\hat{p} = \frac{\sum_{i=1}^{n} Y_i}{n} \tag{1}$$

Let L_i= the likelihood for observation i. What then is the likelihood for the ith vote? For each observation we could get a vote for the proposal with probability p or we could get a vote against the proposal which happens with probability $1 - p$:

$$L_i = p^{Y_i}(1-p)^{1-Y_i} \tag{2}$$

Because we are assuming each of the votes are independent events, the log likelihood is simply the following:

$$lnLn = \sum_{i=1}^{N} [Y_i ln(p) + (1 - Y_i) ln(1 - p)]$$

What value of p maximizes this log likelihood function?

Take partial first derivatives with respect to p to get:

$$\frac{\partial lnL}{\partial p} = 0$$

Solving these simultaneously for p, we get:

$$p = \frac{\sum_{i=1}^{N} Y_i}{N} \tag{3}$$

So in this case our maximum likelihood estimate of p is simply 3/5 because $n = 5$ and the sum of 1,1,1,0,0 is 3.

What is the log-likelihood of $p = .5$?

$$
\begin{aligned}
lnLn &= \sum_{i=1}^{N}(Y_i)ln(p) + \sum_{i=1}^{N}(1 - Y_i)ln(1 - p) \\
&= 1(ln(.5)) + (1 - 1)ln(1 - .5) \\
&= 1(ln(.5)) + (1 - 1)ln(1 - .5) \\
&= 1(ln(.5)) + (1 - 1)ln(1 - .5) \\
&= 0(ln(.5)) + (1 - 0)ln(1 - .5) \\
&= 0(ln(.5)) + (1 - 0)ln(1 - .5) \\
&= -3.466
\end{aligned}
$$

What is the log-likelihood of the MLE, $p = .6$

$$
\begin{aligned}
lnLn &= \sum_{i=1}^{N}(Y_i)ln(p) + \sum_{i=1}^{N}(1 - Y_i)ln(1 - p) \\
&= 1(ln(.6)) + (1 - 1)ln(1 - .6) \\
&= 1(ln(.6)) + (1 - 1)ln(1 - .6) \\
&= 1(ln(.6)) + (1 - 1)ln(1 - .6) \\
&= 0(ln(.6)) + (1 - 0)ln(1 - .6) \\
&= 0(ln(.6)) + (1 - 0)ln(1 - .6) \\
&= -3.365
\end{aligned}
$$

Note the log likelihood for $p = .6$ is greater than the log likelihood for $p = .5$.

The likelihood ratio test comparing $p = .6$ with $p = .5$ generates the following

$$
\begin{aligned}
-2(lnL_{.5} - lnL_{.6}) &= -2(-3.466 - (-3.365)) \\
&= -.20
\end{aligned}
$$

The log likelihoods are not significantly different at n=5 and hence an hypothesized $p = .5$ cannot be rejected.

With 10 people and 6-4 in favor we get a likelihood of -6.73. And with 50 people splitting 30-20 we get a likelihood of -33.65. Note that as the n increases our likelihoods get increasing smaller and the second derivative of the log-likelihood function in the neighbourhood of the MLE is becoming more precise.

Question 3

- Estimate an Ordered Logit and present your results in a proper table

```
. ologit demsat labvote trustgov econ lrself if demsat < 5
```

Table 1: Ordered Logit Results

Labour Vote	0.446*
	(0.211)
Trust in Government	0.242*
	(0.036)
Economic Perceptions	-0.251*
	(0.079)
Left-Right Self-Placement	0.065
	(0.034)
γ_1	-1.735*
	(0.388)
γ_2	0.039
	(0.377)
γ_3	3.079*
	(0.403)

Dependent variable is Satisfaction w/Democracy
Log-Likelihood=-689.304, N=647
LR $\chi^2_4 = 108$, $p \approx 0.000$
Main entries are Ordered Logit coefficients
Standard errors in parentheses
$*\ p < 0.05$, two-tailed

- Write out the probability of being in each category.

There need to be four equations, since there are four categories. Notice, however, that there are only three cut-points (γ's).

$$
\begin{aligned}
Pr(Y=1|X) &= \frac{1}{1+e^{-(-1.735-(0.446LV+0.242T-0.251E+0.065LR))}} \\
Pr(Y=2|X) &= \frac{1}{1+e^{-(0.039-(0.446LV+0.242T-0.251E+0.065LR))}} \\
&- \frac{1}{1+e^{-(-1.735-(0.446LV+0.242T-0.251E+0.065LR))}} \\
Pr(Y=3|X) &= \frac{1}{1+e^{-(3.079-(0.446LV+0.242T-0.251E+0.065LR))}} \\
&- \frac{1}{1+e^{-(0.039-(0.446LV+0.242T-0.251E+0.065LR))}} \\
Pr(Y=4|X) &= 1-\frac{1}{1+e^{-(3.079-(0.446LV+0.242T-0.251E+0.065LR))}}
\end{aligned}
$$

- Calculate the probability of being in each category where: E=2, T=10, LV=1, and LR=3

Pr(Y=1—X)	0.017
Pr(Y=2—X)	0.076
Pr(Y=3—X)	0.589
Pr(Y=4—X)	0.318

```
. dis .446+.242*10-.251*3+.065*3
2.308

. dis invlogit(-1.735-2.308)
.01724225

. dis invlogit(0.036-2.308) - invlogit(-1.735-2.308)
.07622636

. dis  invlogit(3.079-2.308)-invlogit(0.036-2.308)
.59026856

. dis  1-invlogit(3.079-2.308)
.31626283
```

- Create a graph of predicted probability of being in each category using `prgen`

```
. prgen trust, gen(tg) rest(median)
```

```
ologit: Predicted values as trustgov varies from 1 to 10.

     labvote  trustgov      econ    lrself
x=         0         5         3         5

. twoway line tgp1 tgp2 tgp3 tgp4 tgx
```

- Do a Likelihood Ratio test of Labour vote and left-right self-placement coefficients both equal to zero.

```
. ologit demsat labvote trustgov econ lrself
Log likelihood = -689.30488

. ologit demsat trustgov econ if e(sample)
Log likelihood = -692.72309

. dis -2*(-692.72309 - -689.30488)
6.83642

. dis chi2tail(2, 6.83642)
.03277104
```