Multi-level Modeling: Applications

Raymond Duch and Dave Armstrong

University of Oxford
Department of Politics and International Relations
Center for Research Methods in the Social Sciences
Nuffield College

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Why Multi-level Modeling

Why Multi-level Modeling?:

1. Theory suggests that micro-effects vary systematically across contexts – political, economic social, etc.

2. Some examples: Kedar (2005) Anderson (1997) Leoni (2006) van der Brug et al (2007) See special issue of Political Analysis (2005).

These efforts have the following common modeling features:

$$\nu_{ik} \sim Bin(\pi_{ik}) \tag{1}$$

$$logit(\pi_{ik}) = \beta_{0k} + \beta_{1k} X_{ik} + \Sigma_{j=1}^{J_k} \phi_{jk} Z_{jik} \tag{2}$$

where (in the example we are going to develop today):

1. ν_{ik} indicates a vote for the Chief Executive party by voter i in each of k election surveys where $i = 1 ... n_k$
2. X_{ik} are retrospective economic evaluations measured at the individual level
3. Z_{ijk} are other characteristics of individuals that shape self-reported vote choice
4. J_k indicates number of controls associated with each of k elections
5. β_{0k} and β_{1k} describe economic voting in any particular survey and vary from survey to survey
6. C_k is a contextual variable measured at level-2

The Data

1. idea here is to maximize the observations on k so as to have pretty robust estimates of contextual effects
2. example here is start with about 200 voter preference surveys resulting in 163 usable surveys
3. http://wwww.raymondduch.com/economicvoting/firststage/index.htm

The Data

Figure: Description of Individual Country Studies

UK

Study	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Vote											
Vote Intention											
Reported Vote											
Economy											
Newret											
Retnat											
Retnat variant											
Demographics											
Age											
Sex											
Income											
Education											
Religiosity											
Church Attendance											
Region											
Language											
Ethnicity											
Class											
Occupation											
Urban-Rural											
Home Ownership											
Private Employee											
Public Housing											
Union Household											
Partisanship											
Party Attachment-U.S.											
Party Attachment											
Party Feeling											
Party Ratings											
Expected Utility											
Left-Right Self Placement											
Left-Right Distance											
Policies											
Policy Distance											
EU Support											
Canadidate Evaluations											
Candidate Feeling											
Candidate Traits											
PM Popularity Rating											
PM Popularity Feeling											
Canadidate Ratings											
Dissatisfaction											
R-Square	0.27			0.29	0.32	0.29	0.24	0.32	0.37	0.38	
Number of Observations	1172			682	549	764	719	648	718	682	
Type of Study	NES			EB	EB	EB	EB	EB	EB	EB	
Access MNL Results	View			View	View	View	View	View	View	View	
Access Codebook	View			View	View	View	View	View	View	View	

Criteria for Selecting Surveys

1. similarly measured dependent variable: vote choice, support for EU, democratic satisfaction
2. similarly measured theoretically critical independent variable: economic perceptions, left-right identification, Postmaterialism
3. appropriate control variables

Workshop Goals

1. replicate analysis in Duch and Stevenson (2005) and (2007) at wwww.raymondduch.com/economic voting
2. employ subset of D&S data
3. pooled strategy – as reviewed by Professor Snijders
4. two-stage strategy – as presented in Duch and Stevenson

Data and Code

The data (a subset of 34 voter preference surveys from Duch and Stevenson) and code for the MLwiN and R programmes that we will be running are available at:

www.raymondduch.com/multilevelmodeling/data.html

Figure: Description of study variables and links to codebooks/definitions

Study		Belgium 1985	Belgium 1987	Belgium 1988	Belgium 1999	Denmark 1985	Denmark 1987	Denmark 1988	France 1985	France 1987	France 1988	Germany 1985	Germany 1987	Germany 1988	Greece 1985	Greece 1988	Ireland 1985	Ireland 1987
Vote																		
Vote Intention																		
Economy																		
Retnat																		
Demographics																		
Age																		
Male																		
Income																		
Education																		
Religiosity																		
Church Attendance																		
Region																		
Language																		
Ethnicity																		
Class																		
Occupation																		
Urban-Rural																		
Home Ownership																		
Private Employee																		
Public Housing																		
Union Household																		
Expected Utlity																		
Left-Right Self Placement																		
Policies																		
EU Support																		
Policy Distance																		
Candidate Evaluations																		
Candidate Feeling																		
Democratic Satisfaction																		
R-Square		0.31	0.4	0.28	0.16	0.4	0.44	0.34	0.33	0.3	0.37	0.32	0.24	0.34	0.82	0.76	0.14	0.07
Number of Observations		565	492	379	1604	641	586	728	547	558	659	770	689	727	504	420	530	504
Type of Study		EB	EB	EB	CSES	EB	EB	EB	EB	EB	EB	EB	EB	EB	EB	EB	EB	EB

Estimate a Pooled "One-Stage" Multi-level Model

$$logit(\pi_{ik}) = \beta_{0k} + \beta_{1k} X_{ik} + \Sigma_{j=1}^{J_k} \phi_{jk} Z_{jik} \quad (3)$$

$$\beta_{0k} = \gamma_{00} + \gamma_{01} C_k + \omega_{0k} \quad (4)$$

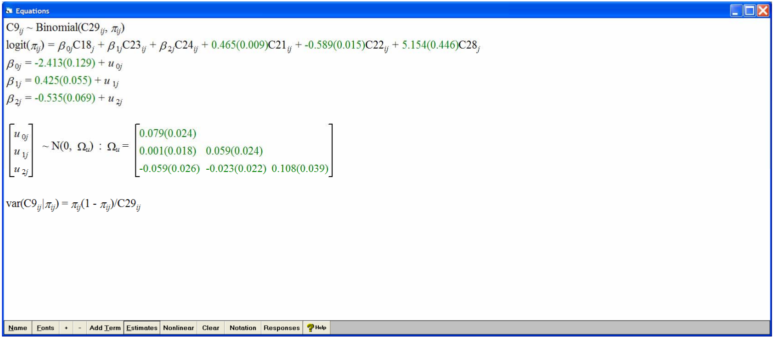
$$\beta_{1k} = \gamma_{10} + \gamma_{11} C_k + \omega_{1k} \quad (5)$$

$$\begin{pmatrix} \omega_{0k} \\ \omega_{1k} \end{pmatrix} \sim N(0, \Omega), \Omega = \begin{pmatrix} \sigma^2_{\omega 0} & \sigma_{\omega 0, \omega 1} \\ \sigma_{\omega 0, \omega 1} & \sigma^2_{\omega 1} \end{pmatrix} \quad (6)$$

MLwiN Data for Replication of Results in Table 2 D&S

```
C9   = 0 (opposition) 1 (chief executive party)
C23 = positive retrospective evaluation dummy
C24 = negative retrospective evaluation dummy
C21 = demeaned Left-Right self identification (1=Left/10=Right)
C22 = interaction of Left-Right self identification with dummy for Left Chief Executive
    party
C28 = average support for the chief executive's party in the survey
C25 = Concentration of Authority (deviated from the study mean)
C26 = Interaction of concentration of authority with positive retrospective evaluation
    Dummy
C27 = Interaction of concentration of authority with negative retrospective evaluation
    Dummy
C29 = constant (1)
C18 = constant (1)
C17 = level 1 unique id number
C8   = level 2 voter preference survey identification code
```

Figure: MLwiN Output for Basic Economic Voting Multi-level Model



"One-Stage" Multilevel Model Results from R

Fixed Effects	Model 1
Economy Better	0.42
	(0.05)
Economy Worse	−0.53
	(0.06)
Left-Right Placement	0.47
	(0.00)
Left-Right Placement × Left Chief Exec	−0.59
	(0.01)
Chief Exec Support	5.22
	(0.43)
Intercept	−2.44
	(0.12)
$\sigma^2_{\omega_1}$	0.07
$\sigma^2_{\omega_2}$	0.06
$\sigma^2_{\omega_3}$	0.10
$\sigma_{\omega_1,\omega_2}$	0.00
$\sigma_{\omega_1,\omega_3}$	−0.05
$\sigma_{\omega_2,\omega_3}$	−0.02

Main entries are logit coefficients.
Standard Errors in parentheses.
N=31286

A "Two-Stage" Modeling Strategy

The advantages of a two-stage strategy are:

1. by avoiding pooling you can increase the number of observations on the context variable
2. well specified models
3. exploratory data analysis

First Stage Estimation for Each Survey

$$\pi_{ikm} = \frac{e^{\hat{\beta}_{1km}(X_{ik}) + \Sigma_{j=1}^{J_k} \hat{\phi}_{jkm} Z_{jik}}}{1 + \sum_{m=1}^{N_{parties}} e^{\hat{\beta}_{1km}(X_{ik}) + \Sigma_{j=1}^{J_k} \hat{\phi}_{jkm} Z_{jik}}} \quad (7)$$

where m indexes party, i indexes individual, j indexes control variables and k indexes surveys.

The Data

Figure: Example of Dutch 1994 Results

The modal category for retnat is 2

The maximum category is 3

The minimum category is 1

Multinomial regression		
	Number of obs =	698
	LR chi2(60) =	580.14
	Prob > chi2 =	0.0000
Log likelihood = -869.9558	Pseudo R2 =	0.2501

vote	RRR	Std. Err.	z	P>|z|	[95% Conf.	Interval]
exleft						
_Iretnat_1	.8453996	.3638151	-0.39	0.696	.3637057	1.965052
_Iretnat_3	.9299575	.3825513	-0.18	0.860	.4152496	2.082653
age	.9836221	.0113278	-1.43	0.152	.9616688	1.006077
male	.8971307	.3091045	-0.32	0.753	.4566421	1.762526
edulevel	1.692483	.3254997	2.74	0.006	1.160969	2.467335
notrelig	.467076	.1660622	-2.14	0.032	.2326778	.9376054
urban	1.476846	.1808652	3.18	0.001	1.161691	1.877498
occupa2	1.618496	.7717652	1.01	0.313	.6356585	4.120969
occupa3	2.382516	1.341695	1.54	0.123	.7901114	7.184279
demsat	2.460696	.6662124	3.33	0.001	1.447446	4.183247
lrself	.7795527	.0938181	-2.07	0.039	.6157501	.9869302
eusup	1.140533	.4037941	0.37	0.710	.5698342	2.282796
d66						
_Iretnat_1	.727626	.2136086	-1.08	0.279	.4092811	1.293584
_Iretnat_3	.8071453	.2364418	-0.73	0.465	.4545737	1.433175
age	.9695737	.0081973	-3.65	0.000	.9536396	.985774
male	.8205655	.1978585	-0.82	0.412	.5115259	1.316312
edulevel	1.340157	.1958723	2.00	0.045	1.006346	1.784695
notrelig	.6165335	.1563864	-1.91	0.057	.3750124	1.013603
urban	.8870917	.0735722	-1.44	0.149	.7540028	1.043672
occupa2	2.693006	.9047309	2.95	0.003	1.394027	5.202396
occupa3	2.637594	1.082383	2.36	0.018	1.180055	5.895403
demsat	.9604605	.2145417	-0.18	0.857	.6199311	1.488044
lrself	1.252968	.1030661	2.74	0.006	1.066405	1.47217
eusup	.813393	.2058959	-0.82	0.415	.4952607	1.335878
cda						
_Iretnat_1	1.165976	.3896794	0.46	0.646	.6056347	2.244754
_Iretnat_3	1.026382	.3551081	0.08	0.940	.5209663	2.022125
age	1.001364	.0090695	0.15	0.880	.983745	1.019298
male	.9738299	.2712245	-0.10	0.924	.5641729	1.680947
edulevel	1.144176	.1935448	0.80	0.426	.821309	1.593966
notrelig	.1291218	.0416475	-6.35	0.000	.0686198	.2429683
urban	.6890904	.0698047	-3.68	0.000	.5650016	.8404322
occupa2	2.45783	.9381718	2.36	0.018	1.163166	5.193519
occupa3	2.890067	1.305579	2.35	0.019	1.192285	7.005445
demsat	.958115	.2431133	-0.17	0.866	.5826834	1.575443
lrself	2.190811	.2120683	8.10	0.000	1.812214	2.648502
eusup	1.285561	.386615	0.84	0.404	.7130258	2.317821
lib						
_Iretnat_1	1.368209	.454809	0.94	0.346	.7131859	2.624835
_Iretnat_3	.7249929	.258738	-0.90	0.368	.3602085	1.459196
age	.987342	.0089914	-1.40	0.162	.9698755	1.005123
male	.7551976	.2126795	-1.00	0.319	.4348549	1.311526
edulevel	1.425402	.2345128	2.15	0.031	1.03251	1.967798
notrelig	.5672886	.1663406	-1.93	0.053	.3193114	1.007845
urban	.9944722	.0970855	-0.06	0.955	.8212852	1.20418
occupa2	7.249337	3.462397	4.15	0.000	2.842831	18.48611
occupa3	9.682602	5.165736	4.26	0.000	3.403064	27.54952
demsat	1.668823	.4193068	2.04	0.042	1.019855	2.730753
lrself	2.76987	.2742458	10.29	0.000	2.281296	3.36308
eusup	.8963845	.2592657	-0.38	0.705	.5085072	1.580125
calvin						
_Iretnat_1	.4952705	.3124597	-1.11	0.265	.1438237	1.705511
_Iretnat_3	.3548959	.2360552	-1.56	0.119	.0963681	1.306979
age	.9364164	.0174567	-3.52	0.000	.9028194	.9712635
male	.825121	.4371377	-0.36	0.717	.292121	2.330626
edulevel	1.555916	.4978591	1.38	0.167	.8310371	2.913078
notrelig	.0166124	.0181744	-3.75	0.000	.0019462	.141798
urban	1.009542	.1996058	0.05	0.962	.6852133	1.487385
occupa2	7.605752	6.509048	2.37	0.018	1.421277	40.70104
occupa3	9.085486	8.741764	2.29	0.022	1.378318	59.889
demsat	.7474556	.3547085	-0.61	0.540	.294879	1.894641
lrself	4.797881	.9827544	7.66	0.000	3.21142	7.168064
eusup	.2057455	.0890902	-3.65	0.000	.0880543	.4807402

(Outcome vote==pvda is the comparison group)
test for all economic coeffcients in all equations are 0:

First Stage Estimation of Economic Vote

$$EV_{ikm} = \Pr(y_{ik} = m_k | X_{ik}, Z_{ijk}) - \Pr(y_{ik} = m_k | X'_{ik}, Z_{ijk}) \quad (8)$$

$$EV_{ikm} = \frac{e^{\hat{\beta}_{1km_k}(X_{ik}) + \Sigma_{j=1}^{J_k} \hat{\phi}_{jkm_k} Z_{jik}}}{1 + \sum_{m=1}^{N_{parties_k}} e^{\hat{\beta}_{1km_k}(X_{ik}) + \Sigma_{j=1}^{J_k} \hat{\phi}_{jkm_k} Z_{jik}}} - \frac{e^{\hat{\beta}_{1km_k}(X'_{ik}) + \Sigma_{j=1}^{J_k} \hat{\phi}_{jkm_k} Z_{jik}}}{1 + \sum_{m=1}^{N_{parties_k}} e^{\hat{\beta}_{1km_k}(X'_{ik}) + \Sigma_{j=1}^{J_k} \hat{\phi}_{jkm_k} Z_{jik}}} \quad (9)$$

where m indexes party, i indexes individual, j indexes control variables and k indexes surveys.

Multinomial Logit Simulation

The Multinomial Logit Simulation goes through the following steps:

1. Makes the categorical variable of interest into 2 dummy variables.
2. estimates a multinomial logit with all the required controls in the model.
3. Simulates 1000 draws from the multivariate normal $\mathcal{N}_k\left(\hat{\beta}, \Sigma_{\hat{\beta}}\right)$
4. Takes the model matrix (the matrix of all independent variables) and multiplies it by the simulated coefficients.
5. Calculates and saves the multinomial logit probabilities in p_{ijm1} (n individuals and 1000 simulations, m categories on the dependent variable)

More MNL Simulation

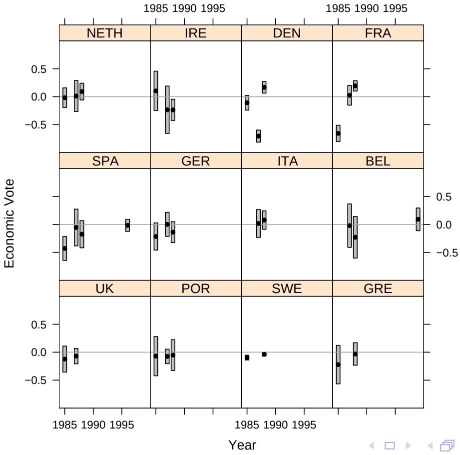
6. Changes the value of the variable of interest (`retnat` here) by one up to a certain boundary condition.
7. Recalculates logit probabilities based on new variable of interest coding and saves them in p_{ijm2}
8. p_{ijm1} - $p_{ijm2} = \delta_{ijm}$
9. $\bar{\delta}_{im} = \left(\frac{1}{1000}\right) \sum_{j=1}^{1000} \delta_{ijm}$
10. Calculates $\sigma^2_{\bar{\delta}_{im}} = \frac{\sum_{j=1}^{1000}\left(\delta_{ijm}-\bar{\delta}_{im}\right)^2}{1000-1}$
11. Calculates $\text{ev}_m = \left(\frac{1}{n}\right) \sum_{i=1}^{n} \bar{\delta}_{im}$
12. Calculates $\sigma_{\text{ev}_m} = \left(\frac{1}{n}\right) \sum_{i=1}^{n} \sigma_{\bar{\delta}_{im}}$
13. Returns ev_m and σ_{ev_m}.

Table: Small Sample of output from R

party	change	sd	country	year
cen.cpp	−0.03	0.047	DEN	1985
cons	−0.107	0.066	DEN	1985
exleft	0.11	0.063	DEN	1985
lib	0.013	0.043	DEN	1985
rad	−0.014	0.051	DEN	1985
socdem	0.028	0.097	DEN	1985
green	0.078	0.047	FRA	1985
pcf-psu	0.115	0.047	FRA	1985
ps−mrg	−0.657	0.073	FRA	1985
rpr	0.231	0.061	FRA	1985
udf	0.234	0.065	FRA	1985
christ	−0.216	0.126	GER	1985
fdp	−0.132	0.106	GER	1985
green	0.086	0.032	GER	1985
soc	0.262	0.119	GER	1985

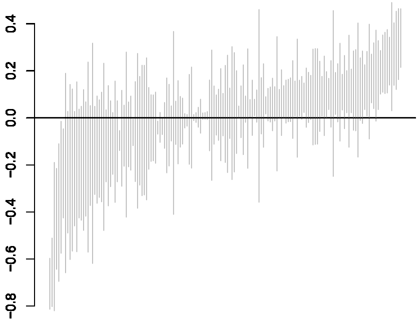
Examining the Data

Figure: Economic Vote of Chief Executive



Examining the Data

Figure: Economic Vote of all Political Parties in Data Set



Second Stage Models

Two-Stage Model:

$$EV_k = \alpha_0 + \alpha_1 C_k + \nu_k \quad (10)$$

One-Stage Multilevel Model:

$$\begin{aligned}
logit(\nu_{ik}) &= \beta_{0k} + \beta_{1k} Worse_{ik} + \beta_{2k} Better_{ik} & (11)\\
&+ \phi_{1k} Ideology_{ik} + \phi_{2k}(Ideology_{ik} \times CE_{Ideology_k}) & (12)\\
\beta_{0k} &= \gamma_0 + \gamma_{01} C_k + \omega_{0k} & \\
\beta_{1k} &= \gamma_1 + \gamma_{11} C_k + \omega_{1k} & (13)\\
\beta_{2k} &= \gamma_2 + \gamma_{11} C_k + \omega_{2k} & (14)
\end{aligned}$$

Comparing Level Two Residuals

In Duch and Stevenson there is a comparison of the one and two stage estimates of the random component of the coefficient for the dummy variable measuring whether respondents indicated "worse economic perceptions".

Estimating survey-level residuals for two-stage strategy

For the sake of generating random effects on "worse economic perceptions" similar to the multilevel logit model, consider the following first-stage model:

$$
\begin{aligned}
y_{ik} &\sim \mathcal{B}(\pi_{ik}) && (15)\\
logit(\pi_{ik}) &= \beta_{0k} + \beta_{1k} Worse_{ik} + \beta_{2k} Better_{ik} && (16)\\
&+ \phi_{1k} Ideology_{ik}
\end{aligned}
$$

Now, we can calculate the following vector of values similar to the random effect residuals on the coefficient of interest:

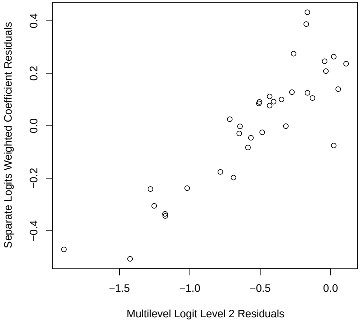
$$
\psi_{1k} = \hat{\beta}_{1k} - \frac{1}{N}\Sigma_{j=1}^{J_k}\hat{\beta}_{1k} n_k \quad (17)
$$

Steps to generate the two random components for comparison

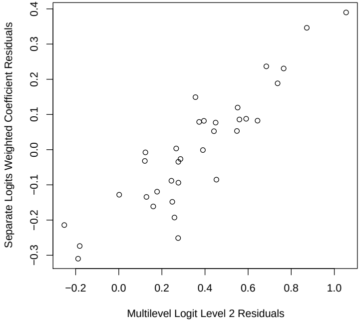
1. estimate ω_{1k} from Equation 13 which is random component for β_{1k}
2. save these in a file
3. estimate the model in Equation 16 for all 34 studies
4. sum the 34 estimates of β_{1k} from Equation 16 multiply them by their sample size and then divide by total observations in all 34 studies – this is the average slope coefficient over all 34 estimates
5. to generate ψ_{1k} in Equation 17 simply subtract this average slope coefficient from each β_{1k} estimated in Equation 16

Comparison of "Residuals" from One and Two Stage Models

Figure: Compare Estimates from One and Two Level



(a) Econ: Worse



(b) Econ: Better

Standard Errors

- Obtaining the appropriate estimates of coefficients and standard errors in the second stage model is of paramount concern. Assume that you have a level 1 model as follows:

$$y_{ik} \sim \mathcal{B}(\pi_{ik}) \quad (18)$$

$$\log\left(\frac{\pi_{ik}}{1-\pi_{ik}}\right) = \alpha + \mathbf{X}\beta + \varepsilon_{ik} \quad (19)$$

- And the second stage model:

$$\hat{y}_k \sim \mathcal{N}(\mu_k, \sigma) \quad (20)$$

$$\mu_k = \gamma_0 + \gamma_1 X_k \quad (21)$$

Where $\hat{y}_k$ is a prediction from the level 1 model.

Why Standard Errors?

- The usual way to treat $\hat{y}_k$ is to take the predicted values and stick them in a predictive model as the dependent variable.
- However, we propose a strategy based on treating these estimates as a severe case of missing data.

$$\bar{Q} = \left(\frac{1}{m}\right)\sum_{j=1}^{m}\beta_{jk} \quad (22)$$

$$B = \left(\frac{1}{m-1}\right)(\beta_{jk} - \bar{Q})(\beta_{jk} - \bar{Q})\prime \quad (23)$$

$$\bar{U} = \left(\frac{1}{m}\right)\sum_{j=1}^{m}\mathbf{V}_j \quad (24)$$

$$T = \bar{U} + \left(1 + \frac{1}{m}\right)B \quad (25)$$

where, $\mathbf{V}_j$ is the variance-covariance matrix of each simulated set of β's, $\bar{Q}$ is the vector of parameter estimates and $\mathbf{T}$ is the total variance-covariance matrix of $\bar{Q}$ (m indexes the simulation).

New Standard Errors

- Our strategy is to simulate each $\hat{y}_k$ from its sampling distribution.
- Then, we get a vector of β's for each simulated vector of $\hat{y}_k$'s.
- Then, we subject these vector of coefficients to the equations above, where we get a better sense of the true sampling variation of in the coefficients once we take account of the error.

Models

Second stage of two-stage model:

$$EV_k = \alpha_0 + \alpha_1 Authority_k + \nu_k \quad (26)$$

One-stage model:

$$\begin{aligned} logit(\pi_{ik}) &= \beta_{0k} + \beta_{1k} Worse_{ik} + \beta_{2k} Better_{ik} & (27) \\ &+ \phi_{1k} Ideology_{ik} + \phi_{2k}(Ideology_{ik} \times CE_{Ideology_k}) & (28) \end{aligned}$$

$$\begin{aligned} \beta_{0k} &= \gamma_{00} + \gamma_{01} Size_k + \gamma_{02} Authority_k + \omega_{0k} \\ \beta_{1k} &= \gamma_1 + \gamma_{11} Authority_k + \omega_{1k} & (29) \\ \beta_{2k} &= \gamma_2 + \gamma_{11} Authority_k + \omega_{2k} & (30) \end{aligned}$$

Multilevel Model Results

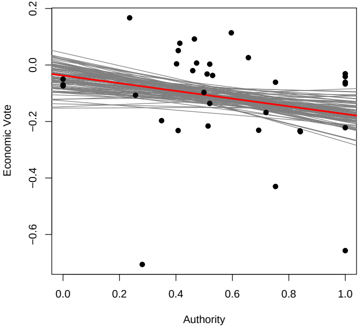
Fixed Effects	Model 1	Model 2
Economy Better	0.42	0.45
	(0.05)	(0.05)
Economy Worse	−0.53	−0.59
	(0.06)	(0.06)
Left-Right Placement	0.47	0.47
	(0.00)	(0.00)
Left-Right Placement × Left Chief Exec	−0.59	−0.59
	(0.01)	(0.01)
Chief Exec Support	5.22	4.19
	(0.43)	(0.40)
Authority		0.99
		(0.22)
Authority × Better		0.49
		(0.26)
Authority × Worse		−0.76
		(0.31)
Intercept	−2.44	−2.11
	(0.12)	(0.12)
$\sigma^2_{\omega_1}$	0.07	0.03
$\sigma^2_{\omega_2}$	0.06	0.05
$\sigma^2_{\omega_3}$	0.10	0.08
$\sigma_{\omega_1,\omega_2}$	0.00	−0.01
$\sigma_{\omega_1,\omega_3}$	−0.05	−0.02
$\sigma_{\omega_2,\omega_3}$	−0.02	−0.01

Main entries are logit coefficients, Standard Errors in parentheses.
N=31286

Second-stage Model

$$EV_k = \alpha_0 + \alpha_a Authority_k + \upsilon_k \quad (31)$$

	Intercept	Authority
α	-0.037	-0.136
σ_α	0.070	0.112
$\tilde{\sigma}_\alpha$	0.092	0.146
$\tilde{\sigma}_\alpha/\sigma_\alpha$	1.305	1.310



Where σ_α is the OLS SE and $\tilde{\sigma}_\alpha$ is the adjusted OLS SE