

University of Oxford

Formal Analysis: Lecture Four

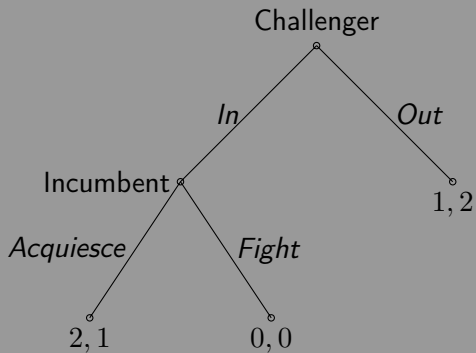
Raymond M. Duch

Nuffield College Oxford
www.raymondduch.com
@RayDuch

Hilary 2012

Extensive Form

- Extensive form games have four components:



└ Extensive Form Games

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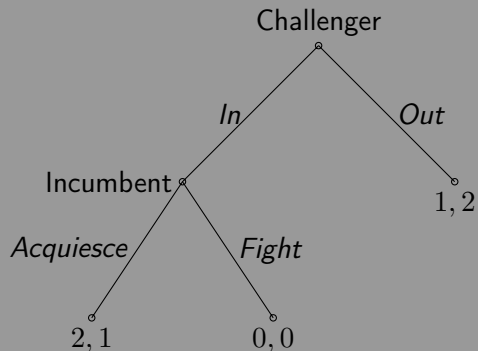
Normal form games assume simultaneous action. Most real world situations aren't characterized by simultaneous actions. How do we incorporate sequential actions into game theory?

Explain players, terminal histories, player function, and preferences.

Each players actions are defined by the histories and player function.

Extensive Form

- Extensive form games have four components:
 - ▶ a set of players



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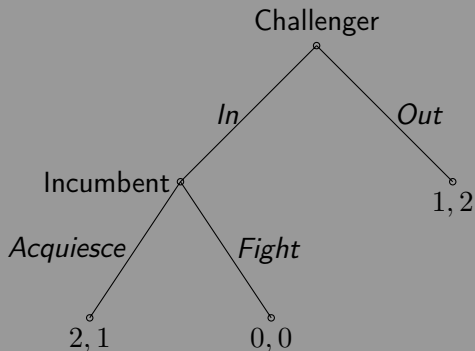
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 - ▶ a set of players
 - ▶ a set of sequences (terminal histories)



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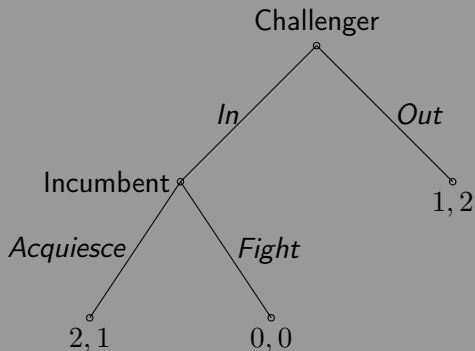
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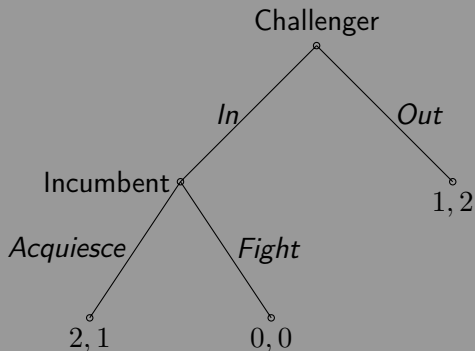
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 - ▶ preferences over the set of terminal histories



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Extensive Form

Finite Horizon: Terminal histories aren't infinitely long.

A game is finite if no terminal history is infinitely long and there are finitely many terminal histories.

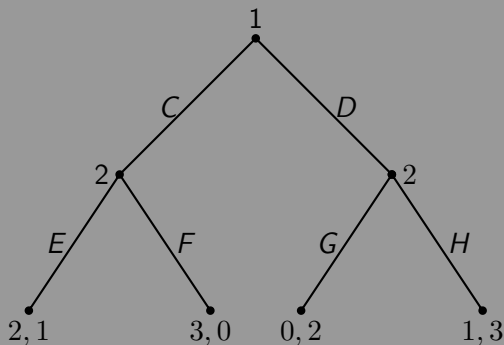
An extensive form game with perfect information is a model of a situation in which the players know all the actions taken previously and always moves alone.

Extensive Form

Solving extensive form games

Strategy: A function that assigns an action to each *history* in which it is the player's turn

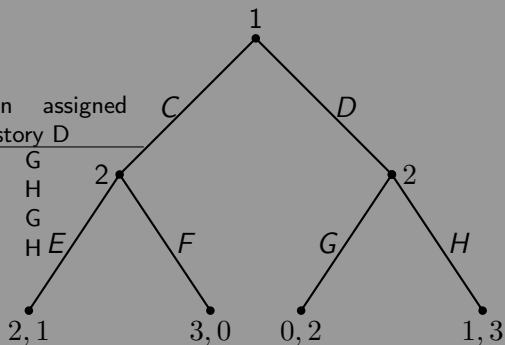
Important: Specifies what a player will do for every contingency!



Extensive Form

	Action assigned to history C
Strategy 1	E
Strategy 2	E
Strategy 3	F
Strategy 4	F

	Action assigned to history D
	G
	H
	G
	H



Nash Equilibrium

- Outcome of game is determined by the action taken along the path of play (terminal history).
- We refer to outcomes as $O(s)$

Definition (Nash Equilibrium)

The strategy profile s^* is a NE if for all players and every strategy r_i the terminal history $O(S^*)$ is at least as good as those generated by any deviation.

$$u_i(O(s^*)) \geq u_i(O(r_i, s_{-i}^*)), \forall r_i \text{ of player } i$$

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Definition

$$u_i(\alpha^*) \geq u_i(\alpha_i, \alpha_{-i}^*), \forall \alpha_i \in A_i$$

Nash Equilibrium

- Thus, one way to find the NE is to construct a corresponding strategic form game
- Challenger vs. Incumbent game

		Incumbent	
		<i>Acquiesce</i>	<i>Fight</i>
Challenger	<i>In</i>	2, 1	3, 0
	<i>Out</i>	1, 2	1, 2

└ Extensive Form Games

└ Nash Equilibrium

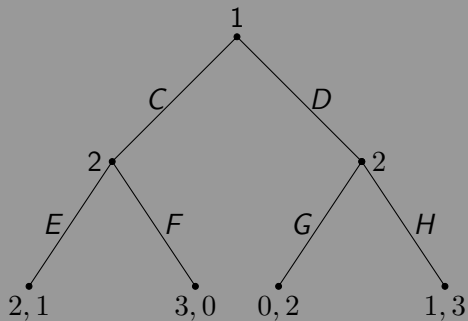
Nash Equilibrium

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- Challenger vs. Incumbent game

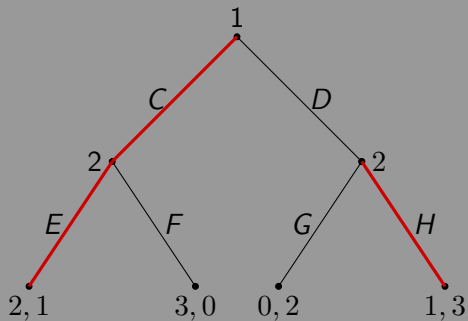
		Incumbent	
		Accommodate	Fight
Challenger	In	2, 1	3, 0
	Out	1, 2	1, 2

Corresponding strategic form game will tend to be larger than we think!
What about the game in the previous slide?

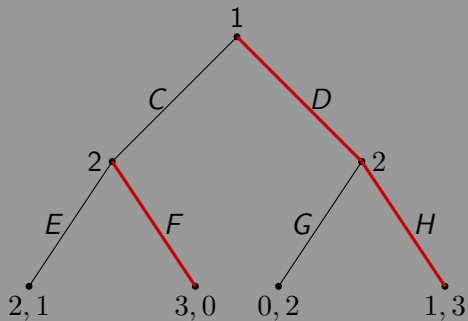
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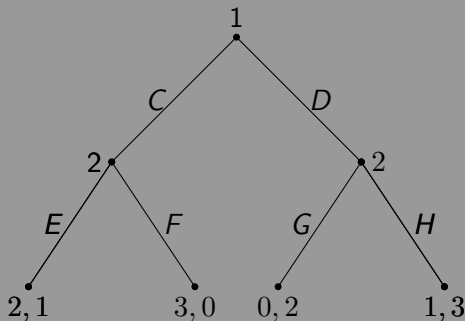


Nash Equilibria



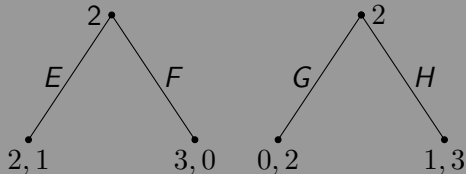
Subgame Perfect Equilibrium

- A player's actions are optimal in every subgame of the game. A subgame $\Gamma(h)$ is the game following any history. The whole game is a subgame of it self. The rest is a **proper subgame**.
- Every subgame perfect equilibria is a NE but not vice versa. A subgame perfect equilibrium is a strategy profile that induces a NE in every subgame of the game.



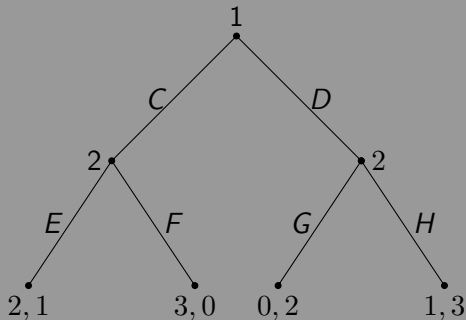
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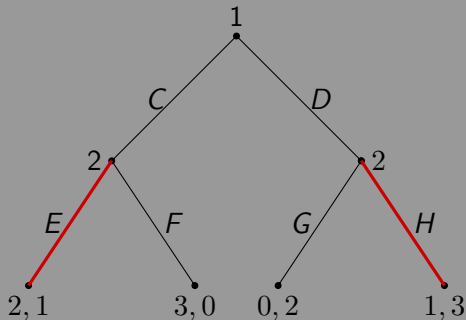
Backwards induction in finite horizon games

- Start at the bottom of the tree (the game with the longest history or with the game with the least length)



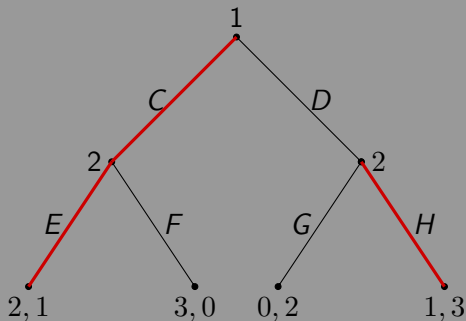
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Homework Assignment

Question Numbers from Osborne:

- 156.2
- 161.1
- 177.2pt