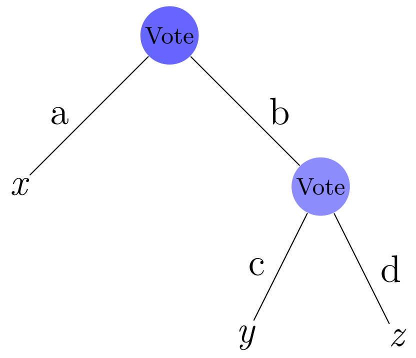
Committee decision-making

How do committees decide? How do decision making procedures influence outcomes?

Assume a set of alternatives and a set of committee members. Simple if two alternatives. If three alternatives, x, y, z, we might imagine a process such as:



We describe processes such as these as *binary agendas* as at each stage of voting the choice is between two options. E.g., at the first stage we consider whether we want to adopt x or not. The number of terminal histories is at least the number of alternatives and each alternative is associated with at least on terminal history.

'The auxiliary game' – simply a device to describe the game formally.

Any binary agenda has a lot of subgame perfect equilibria – but the set of subgame perfect equilibria when weakly dominated strategies are eliminated is much smaller.

'Sophisticated voting outcome' – i.e., committee members look down the game tree. Which alternatives can be sophisticated outcomes?

If Condorcet winner exist it will be the sophisticated voting outcome. The logic - doesn't matter where x appears in the tree.

What if no Condorcet winner? Consider the figure – if x is to be the outcome it must beat y and z but not necessarily 'directly'. It is sufficient to beat on directly and the other indirectly – i.e., $x \succeq y \succeq z$.

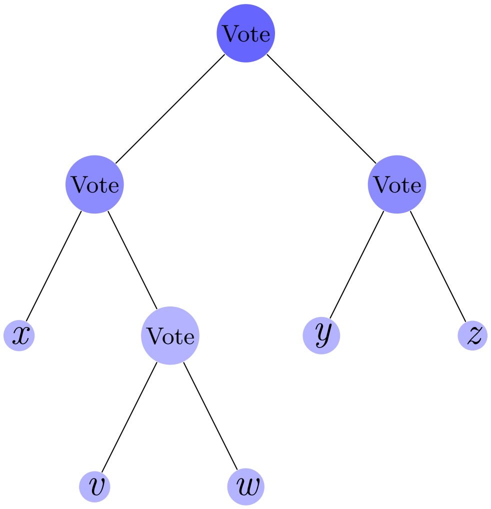
The Top Cycle Set The set of alternatives that x beats indirectly, i.e., x beats y if we can construct a chain of alternatives $z_1, z_2, z_3, \ldots z_k$ such that

x beats z_1, z_1 beats z_2, …and z_k beats y.

Any outcome must be in the top cycle set. But is there an agenda such that x (in the TCS) is the sophisticated voting outcome?

Exercise 221.1 $P1 : x > y > v > w > z$, $P2 : z > x > v > w > y$, $P3 : y > z > w > v > x$

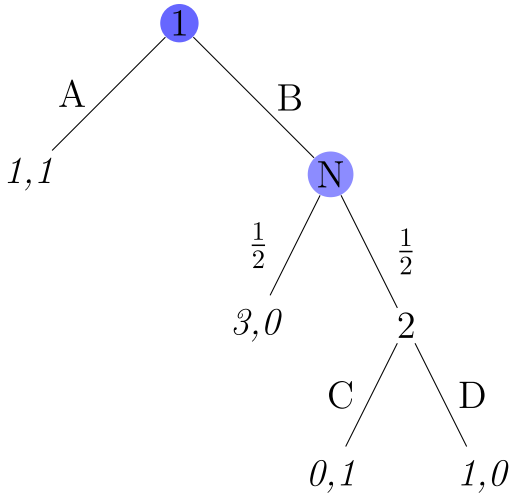
Top cycle set: $x > y > z > x$, x, y, z beat v, $v > w$ and w beats nothing.



Switch y and x to get y. switch z and x to get z

Exogenous uncertainty

We can modify the game to include uncertainty – consider ‘Nature’ a player that takes random actions. Thus, we allow the player function to assign chance or ‘Nature’ to some histories. Probabilities of each action by Nature are given. The players’ preferences are defined over the lotteries of outcomes.



At the last stage (longest history), P2 prefers C to D. P1 obtains 1 if she chooses A and if she chooses B she obtains $3 * \frac{1}{2} + 0 * \frac{1}{2} = \frac{3}{2}$ and thus prefers B. The subgame perfect equilibrium is (B, C).

Exercise 227.3 A duel, each hits with probability p_i. Two equilibria.

(N,N) Clearly an eq.

(S,S) Suppose the probability of surviving in this strategy profile is Π_i. If P1 deviates in any given period then she might have have hit P2 with probability p_1 and she may survive the period if P2 misses (with probability $1-p_2$). Thus the probability of surviving the the first round is $1-p_2$, i.e., the player reaches an identical subgame with probability $1-p_2$. Thus, choosing not to shoot the probability of survival is $(1-p_2)\Pi_1$

Repeated Games: The Prisoner's Dilemma

What if actors interact repeatedly?

- You care about the future
- Past actions may affect future behaviour of players

Consider the 'Grim Trigger Strategy':

C as long as the other player(s) cooperate

D forever if a player has defected

- Note – strategy defines actions in any history

		Player 2	
		C	D
Player 1	C	$2, 2$	$0, 3$
	D	$3, 0$	$1, 1$

Given that player 1 uses this strategy, player 2 essentially has two choices. To *cooperate* forever and receive 2 in each period (2,2,2,. . .) or to *defect* and receive a higher period in the period and a lower payoff ever after (3,1,1,1,. . .). If the player is *patient* then he would prefer cooperation.

Formalize this argument and answer several questions:

- How patient must the players be?
- Are other NE outcomes possible and, if so, what are they?
- Does the logic survive subgame perfection?
- Is the grim trigger strategy too strong?
- Does this argument extend to other games?

Discounted Sum: Captures the notion of patience. Each player has a payoff function u_i and a discount factor $\delta \in [0, 1]$ such that the discounted sum of the action profile $(a^1, a^2, \ldots, a^T)$ equals

$$u_i(a^1) + \delta_i u_i(a^2) + \delta_i^2 u_i(a^3) + \ldots + \delta_i^{T-1} u_i(a^T) = \sum_{t=1}^{T} \delta^{t-1} u_i(a^t) \tag{1}$$

Working with sequences

Consider a geometric series

$$U = x + \delta x + \delta^2 x + \ldots + \delta^T x \tag{2}$$

multiply both sides by: δ

$$\delta U = \delta x + \delta^2 x + \delta^3 x + \ldots + \delta^{T+1} x. \tag{3}$$

Now subtract δU from U:

$$\begin{aligned} U - \delta U &= x + \delta x + \delta^2 x + \delta^3 x + \ldots + \delta^T x \\ &\qquad -\delta x - \delta^2 x - \delta^3 x - \delta^4 x - \ldots - \delta^{T+1} x \\ &= x - \delta^{T+1} x \\ \Longrightarrow & \\ (1-\delta)U &= x - \delta^{T+1} x \\ \Longrightarrow & \\ U &= \frac{x(1-\delta^{T+1})}{1-\delta} \end{aligned} \quad \begin{aligned} (4) \\ \\ \\ \\ (5) \\ \\ (6) \end{aligned}$$

Sometimes we have infinitely long sequences/games. Then δ^{T+1} goes to zero in the limit because δ is assumed to be less than 1 (but bigger than zero). Thus, for an infinite series U reduces to:

$$x + \delta x + \delta^2 x + \ldots = \frac{x}{1-\delta}. \tag{7}$$

Another useful thing is the *discounted average*. We know from the above that the value of getting c in each period is $V = \frac{c}{1-\delta}$. Then $V(1-\delta) = c$, that is, $V(1-\delta)$ equals the per period payoff. Suppose the individual is indifferent between getting c in each period and a stream of payoffs $(x^1, x^2, \ldots)$, i.e, $V = \sum_{t=1}^{\infty} \delta^{t-1} w^t$. Because V is equivalent to getting c in each period we refer to $(1-\delta)V = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} w^t$ as the discounted average. The idea is not that this is the average payoff in each period *post discounting* but rather that it is equivalent to getting c in each period.

For example, suppose $\delta = \frac{1}{2}$. Consider the payoff streams (4,4,4) and (5,2,4). The discounted sum of (4,4,4) equals $4 + 4\frac{1}{2} + 4\frac{1}{4} = 7$ and the discounted sum of (5,2,4) equals $5 + 2\frac{1}{2} + 4\frac{1}{4} = 7$. Thus, the two payoff streams yield the same discounted payoff. The average (un-discounted) per period payoff in the latter case is $\frac{11}{3}$. However, taking the discounting into account, it "averages out" to receiving a payoff of 4 in each period.

Repeated Games

Let G be a strategic game. Consists of:

- set of players N
- terminal histories that are sets of sequences of action profiles in G
- a player function that assigns all players to all histories
- the set of actions after any history is the set of histories in the G
- each player evaluates each terminal history $(a^1, \ldots, a^T)$ according the discounted average.

Finitely Repeated Prisoner's Dilemma

I.e., $T < \infty$. What are the NE?

- Suppose one player's strategy is to always choose D?
- Every NE generates the same outcome path.
- A deviation from C to D in the last period that either player chooses C cannot be punished.

Infinitely Repeated Prisoner's Dilemma

Describing strategies.

Grim Trigger: $s(\emptyset) = C$, and then C if $(a_j^1, \ldots, a_j^t) = (C, \ldots)$.

States: We can also think in terms of states and draw diagrams. Examples, *grim trigger strategy, three period punishment, tit-for-tat.*

Some NE

Grim Trigger. If both use grim trigger then (C, C) in each period. The average discounted payoff is then 2.

Is this eq.? If player two deviates from strategy she must choose D in some period. Player responds with D forever to which D is the best response. The stream of payoffs starting with the deviation is then (3,1,1,...). The discounted average payoff is then

$$(1-\delta)(3+\delta+\delta^2+\ldots) = (1-\delta)(3+\frac{\delta}{1-\delta}) = 3(1-\delta)+\delta \tag{8}$$

Thus player 2 cannot increase her payoff if: $3(1-\delta)+\delta \leq 2$ or $\delta \geq \frac{1}{2}$

Limited Punishment Punishment for k periods. Suppose player 1 uses the strategy. If the strategy is not a best response for player 2 then she chooses D at some point. (If it is the discounted average is 2). If the best response yields a greater payoff it must do so from the period t (deviation occurs) until period $t+k+1$. The payoff from the deviation is:

$$(1-\delta)(3+\delta+\delta^2+\ldots+\delta^k) = 3(1-\delta)+\delta(1-\delta^k) \tag{9}$$

Following the same strategy as player 1, player 2 yields:

$$(1-\delta)(2+2\delta+2\delta^2+\ldots+2\delta^k) = 2(1-\delta^{k+1}). \tag{10}$$

Comparing the discounted average payoffs we then have that cooperating is a best response to itself iff:

$$\begin{aligned} 2(1-\delta^{k+1}) &\geq 3(1-\delta)+\delta(1-\delta^k) && (11)\\ 2-2\delta^{k+1} &\geq 3-3\delta+\delta-\delta^{k+1} && (12)\\ 0 &\geq \delta^{k+1}-2\delta+1 && (13) \end{aligned}$$

If k=1, then no discount factor is sufficient, if k=2 then $\delta \geq .62$ is enough etc.

Tit-for-tat: Again assume player one plays tit-for-tat. There are two types of deviations. i) Player 2 could play D forever and ii) Revert to C after deviation (i.e., alternate forever).

If player 2 alternates the payoffs are (3,0,3,0,. . .) – the discounted average is $(1-\delta)\frac{3}{1-\delta^2}$. To see where this comes from note that the payoff is equal to:

$$U = x+\delta^2 x+\delta^4 x+\ldots \tag{14}$$

Multiplying both sides by δ^2 (above we used just δ – this is because we want the terms to cancel out when we subtract the multiplied series from the original one):

$$\delta^2 U = \delta^2 x+\delta^4 x+\delta^6 x+\ldots \tag{15}$$

Subtracting $\delta^2 U$ from U we then have:

$$U-\delta^2 U = x \tag{16}$$

and, finally:

$$U = \frac{x}{1-\delta^2} \tag{17}$$

If she defects forever then the payoffs are $(3, 1, 1, \ldots)$ and the discounted average is $3(1-\delta)+\delta = 3-2\delta$. So tit-for-tat is a best response to itself if:

$$2 \geq \frac{3}{1-\delta^2} \text{ and } 2 \geq 3-2\delta \tag{18}$$

or when $\delta \geq \frac{1}{2}$.

Feasible payoffs

- Show set of feasible discounted average payoffs – explain idea.
- NE discounted average payoffs.

tricky but fairly easy if *delta* is close to 1.

can't be worse than (D, D) forever.

for any payoff better than (D, D) there exists a strategy for δ close to 1.

one-deviation property

- no player can increase her payoff by changing her action of the start of any subgame in which she is the first-mover, given the other players' strategies and the rest of her strategy
- draw figure 438.1
- a subgame perfect equilibrium iff it satisfies the one-deviation property.

Subgame perfect equilibria

Grim Trigger: No, what happens if player 1 picks D in first period? Is it still optimal to pick C in first period for player 2?