

Formal Analysis: Lecture 4

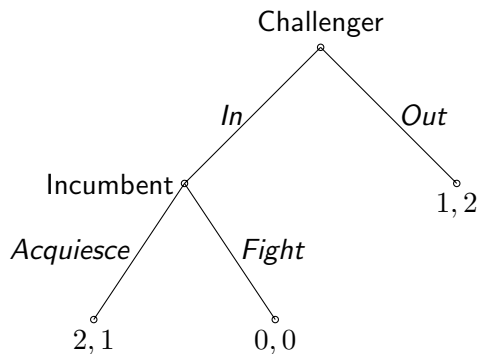
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Hilary 2011

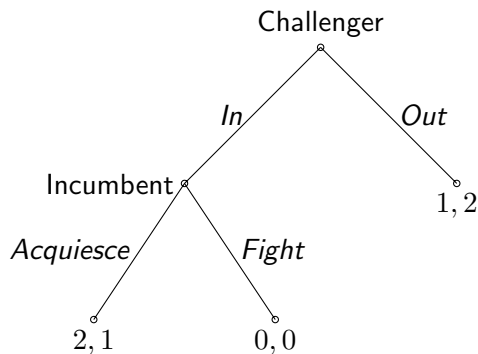
Extensive Form

- Extensive form games have four components:



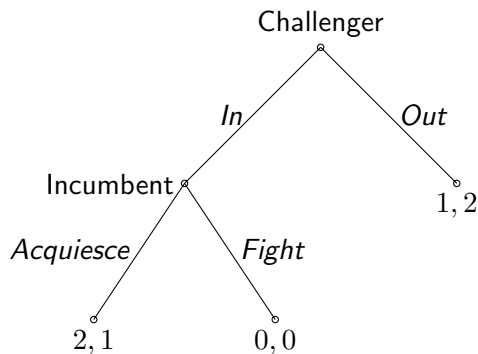
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 - ▶ a set of players



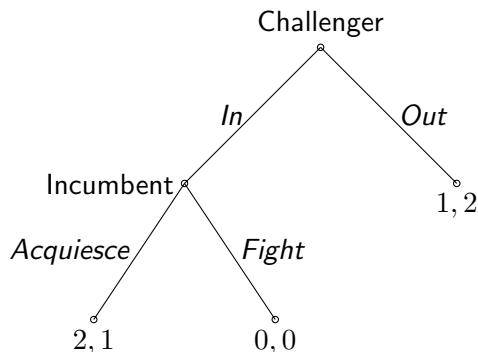
Extensive Form

- Extensive form games have four components:
 - ▶ a set of players
 - ▶ a set of sequences (terminal histories)



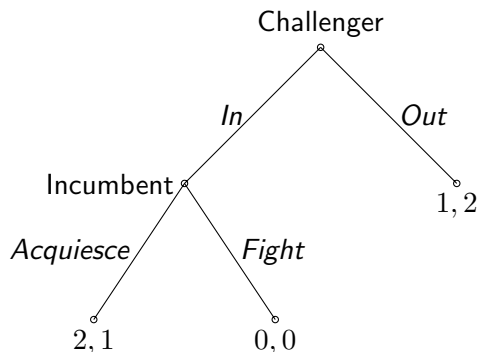
Extensive Form

- Extensive form games have four components:
 - ▶ a set of players
 - ▶ a set of sequences (terminal histories)
 - ▶ a player function that assigns a player to every proper subsequence of some terminal history



Extensive Form

- Extensive form games have four components:
 - ▶ a set of players
 - ▶ a set of sequences (terminal histories)
 - ▶ a player function that assigns a player to every proper subsequence of some terminal history
 - ▶ preferences over the set of terminal histories



Extensive Form

Finite Horizon: Terminal histories aren't infinitely long.

A game is finite if no terminal history is infinitely long and there are finitely many terminal histories.

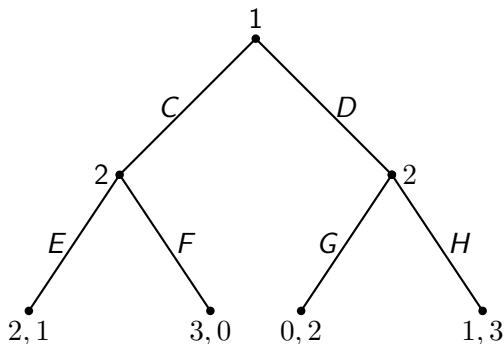
An extensive form game with perfect information is a model of a situation in which the players know all the actions taken previously and always moves alone.

Extensive Form

Solving extensive form games

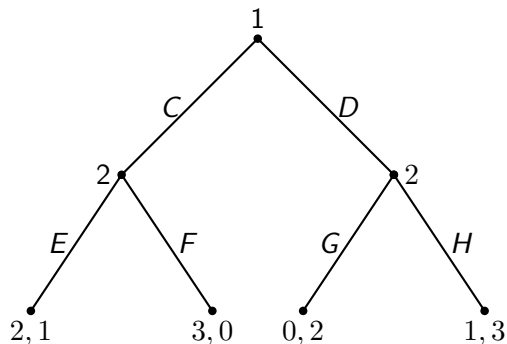
Strategy: A function that assigns an action to each *history* in which it is the player's turn

Important: Specifies what a player will do for every contingency!



Extensive Form

	Action assigned to history C	Action assigned to history D
Strategy 1	E	G
Strategy 2	E	H
Strategy 3	F	G
Strategy 4	F	H



Nash Equilibrium

- Outcome of game is determined by the action taken along the path of play (terminal history).
- We refer to outcomes as $O(s)$

Definition (Nash Equilibrium)

The strategy profile s^* is a NE if for all players and every strategy r_i the terminal history $O(S^*)$ is at least as good as those generated by any deviation.

$$u_i(O(s^*)) \geq u_i(O(r_i, s_{-i}^*)), \forall r_i \text{ of player } i$$

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Definition

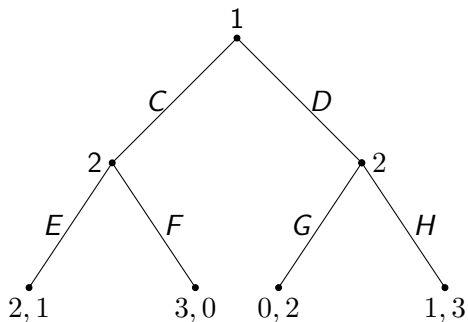
$$u_i(\alpha^*) \geq u_i(\alpha_i, \alpha_{-i}^*), \forall \alpha_i \in A_i$$

Nash Equilibrium

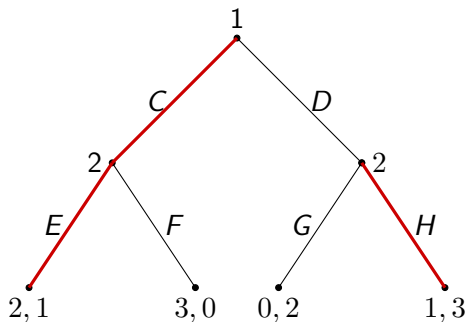
- Thus, one way to find the NE is to construct a corresponding strategic form game
- Challenger vs. Incumbent game

		Incumbent	
		<i>Acquiesce</i>	<i>Fight</i>
Challenger	<i>In</i>	2, 1	3, 0
	<i>Out</i>	1, 2	1, 2

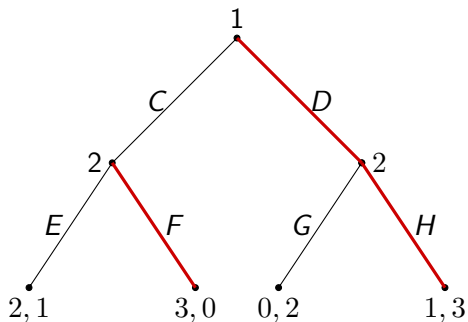
Nash Equilibria



Nash Equilibria

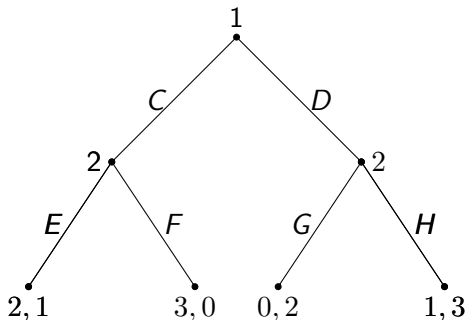


Nash Equilibria



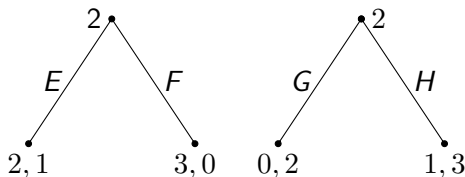
Subgame Perfect Equilibrium

- A player's actions are optimal in every subgame of the game. A subgame $\Gamma(h)$ is the game following any history. The whole game is a subgame of it self. The rest is a **proper subgame**.
- Every subgame perfect equilibria is a NE but not vice versa. A subgame perfect equilibrium is a strategy profile that induces a NE in every subgame of the game.



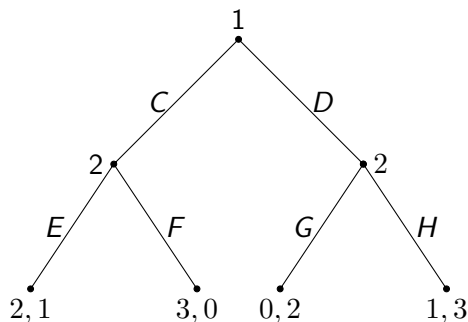
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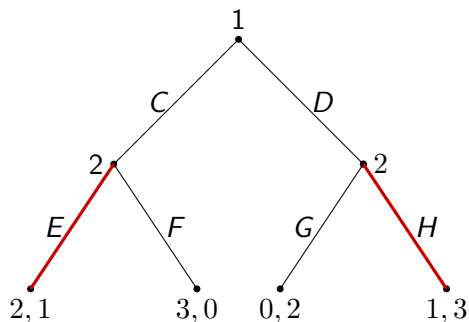
Backwards induction in finite horizon games

- Start at the bottom of the tree (the game with the longest history or with the game with the least length)



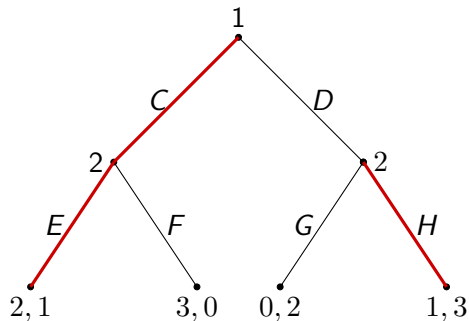
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Backwards induction in finite horizon games

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Homework Assignment

Question Numbers from Osborne:

- 156.2
- 161.1
- 177.2pt