Formal Analysis: Lecture 8

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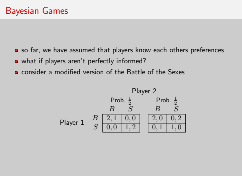
Bayesian Games

- so far, we have assumed that players know each others preferences
- what if players aren't perfectly informed?
- consider a modified version of the Battle of the Sexes

Player 2

		Prob. $\frac{1}{2}$		Prob. $\frac{1}{2}$	
		B	S	B	S
Player 1	B	$2,1$	$0,0$	$2,0$	$0,2$
	S	$0,0$	$1,2$	$0,1$	$1,0$



Normal form games assume simultaneous action. Most real world situations aren't characterized by simultaneous actions. How do we incorporate sequential actions into game theory?

Explain players, terminal histories, player function, and preferences.

Each players actions are defined by the histories and player function.

Bayesian Games

Player 2

Player 1	Prob. $\frac{1}{2}$: B	Prob. $\frac{1}{2}$: S	Prob. $\frac{1}{2}$: B	Prob. $\frac{1}{2}$: S
B	$2, 1$	$0, 0$	$2, 0$	$0, 2$
S	$0, 0$	$1, 2$	$0, 1$	$1, 0$

- there two 'states' – player 2 'likes' or doesn't 'like' player 1
- there are two 'types' of player 2
- player 2 can calculate expected utilities given strategies of each types

	(B, B)	(B, S)	(S, B)	(S, S)
B	2	1	1	0
S	0	$\frac{1}{2}$	$\frac{1}{2}$	1

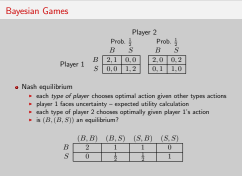
Bayesian Games

Player 2

		Prob. $\frac{1}{2}$ B	Prob. $\frac{1}{2}$ S	Prob. $\frac{1}{2}$ B	Prob. $\frac{1}{2}$ S
Player 1	B	$2,1$	$0,0$	$2,0$	$0,2$
	S	$0,0$	$1,2$	$0,1$	$1,0$

- Nash equilibrium
  - each *type of player* chooses optimal action given other types actions
  - player 1 faces uncertainty – expected utility calculation
  - each type of player 2 chooses optimally given player 1's action
  - is $(B,(B,S))$ an equilibrium?

	(B,B)	(B,S)	(S,B)	(S,S)
B	2	1	1	0
S	0	$\frac{1}{2}$	$\frac{1}{2}$	1



First, consider 1's strategy – if player 2 opts for (B,S) then the expected payoff to B is greater than S (see table).

Second, *meet*-type is happy with (B,B) and *avoid*-type is happy with (B,S).

Bayesian Games

		Prob. $\frac{1}{2}$		Prob. $\frac{1}{2}$	
		B	S	B	S
Prob. $\frac{2}{3}$	B	$2,1$	$0,0$	$2,0$	$0,2$
	S	$0,0$	$1,2$	$0,1$	$1,0$
		B	S	B	S
Prob. $\frac{1}{3}$	B	$0,1$	$2,0$	$0,0$	$2,2$
	S	$1,0$	$0,2$	$1,1$	$0,0$

- types, states & signals
  - each player has two types: y, n
  - four states: yy, yn, ny, nn
  - each player receives a signal that reveals his own type
    - P1: $\tau_1(yy) = \tau_1(yn) = y_1$ & $\tau_1(ny) = \tau_1(nn) = n_1$
  - Consider whether each of the four players strategies are optimal. Consider $((B, B), (B, S))$

Bayesian Games

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	S	$0,0$	$1,2$	$0,1$	$1,0$
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Expected payoff as in the above table.

Type n_1

	(B,B)	(B,S)	(S,B)	(S,S)
B	0	1	1	2
S	1	$\frac{1}{2}$	$\frac{1}{2}$	0

Type y_2

	(B,B)	(B,S)	(S,B)	(S,S)
B	1	$\frac{2}{3}$	$\frac{1}{3}$	0
S	0	$\frac{2}{3}$	$\frac{4}{3}$	2

Type n_2

	(B,B)	(B,S)	(S,B)	(S,S)
B	0	$\frac{1}{3}$	$\frac{2}{3}$	1
S	2	$\frac{4}{3}$	$\frac{2}{3}$	0

Can information hurt?

	Prob. $\frac{1}{2}$	:P1:	Prob. $\frac{1}{2}$
	Prob. $\frac{1}{2}$	:P2:	Prob. $\frac{1}{2}$

	L	M	R
T	$1, 2\varepsilon$	$1, 0$	$1, 3\varepsilon$
B	$2, 2$	$0, 0$	$0, 3$

	L	M	R
T	$1, 2\varepsilon$	$1, 3\varepsilon$	$1, 0$
B	$2, 2$	$0, 3$	$0, 0$

- $0 < \varepsilon < \frac{1}{2}$
  - P2 chooses L: $2\varepsilon > \frac{3}{2}\varepsilon$ and $2 > \frac{3}{2}$
  - P1's best response to L is B
- now, suppose P2 receives signal $\tau(\omega_1) \neq \tau(\omega_2)$
  - now R dominates if in state 1, M if state 2
  - T is best response to M and R

Can information hurt?

Prob. $\frac{1}{2}$:P1: Prob. $\frac{1}{2}$
Prob. $\frac{1}{2}$:P2: Prob. $\frac{1}{2}$

	L	M	R
T	$1, 2\varepsilon$	$1, 0$	$1, 3\varepsilon$
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	L	M	R
T	$1, 2\varepsilon$	$1, 3\varepsilon$	$1, 0$
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Adverse selection (282.3)

- Firm A taking over firm T
  - A doesn't know value of T: equal probability over each dollar value $\{0, 1, \ldots, 100\}$
  - Value of T 50% greater under A
  - A bids y and true value of T is x
  - A's payoff is $\frac{3}{2}x - y$ and T's payoff is y if offer is accepted and x if rejected
- A's action is a bid y
- T's is a threshold for accepting an offer
  - States: possible values of firm T
  - Actions: Set of possible bids (positive numbers) for A and set of possible thresholds
  - Signals: T gets a different signal for each state, A receives the same signal in each state
  - Beliefs: A assigns equal prob. to each state, T assigns prob. 1 to state indicated by signal

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Consider T. T only accepts if $y > x$.
Firm A. If it bids y then T accepts if $y > x$. Thus the average value of a firm that excepts is $\frac{q(y)}{2}$ if $y < 100$ and 50 if $y > 100$.
A's payoff then equals $\frac{3}{2}\frac{q(y)}{2} - y$ (or $50\frac{3}{2} - y$). Both are always negative so the only equilibrium is an offer of 0 and an acceptance threshold x.