

Formal Analysis: Lecture 7: Bargaining Models

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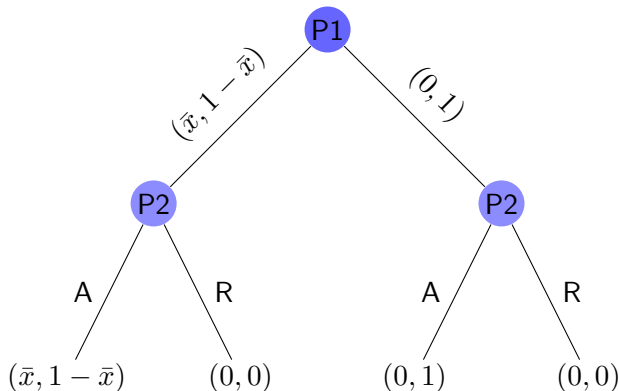
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Ultimatum Game

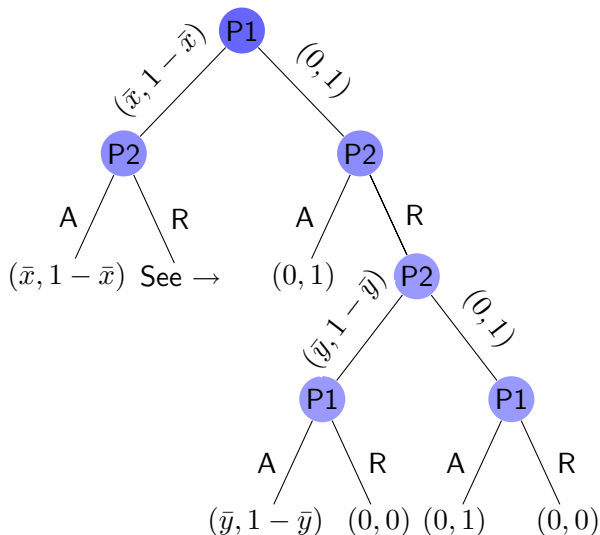
Simple form of bargaining

Player 1 proposes a division of pie, player 2 accepts or rejects. If player 2 rejects both players get 0.

Sequence of actions: Let \bar{x} be any proposal x s.t. $x > 0$.



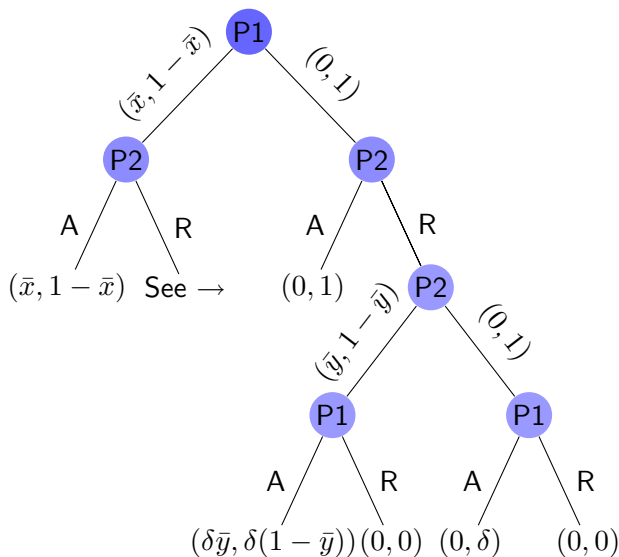
Ultimatum with counterproposal



Equilibria in Ultimatum Game with Counterproposal

Eq. Now player 2 gets the whole pie! What are the equilibria?

Ultimatum Game + Response + Discounted Payoffs



Rubinstein Bargaining Model

- **Players:** $N = \{1, 2\}$
- **Terminal Histories:** Every sequence $(x^1, N, x^2, N, \dots, x^t, Y)$
- **Player Function:** Players alternate
- **Preferences:** $u_i = \delta_i x_i^t$

How do we find the equilibrium?

- All subgames are identically \rightarrow Stationary game
- Stationary strategies seem reasonable, i.e., because the game looks identical after every history it would seem to make sense if the players take the same action after each history.
- A strategy in the game is then a 'rule' specifying what to propose and which offers to accept.

How do we find the equilibrium?

Write the strategies:

$$P1 : x^* \text{ \& A if } y_1 \geq y_1^* \quad (1)$$

$$P2 : z^* \text{ \& A if } w_2 \geq w_2^* \quad (2)$$

Now, suppose $x_2^* > w_2^*$. Player 1 can obviously do better by offering less. Thus, $x_2^* = w_2^*$. Similarly we can argue that $z_2^* = y_2^*$

Then the strategies can be rewritten:

$$P1 : x^* \text{ \& A if } y_1 \geq y_1^* \quad (3)$$

$$P2 : y^* \text{ \& A if } x_2 \geq x_2^* \quad (4)$$

How do we find the equilibrium?

- Consider player 2's actions under these strategies.
 - ▶ If player 2 rejects then her strategy is to propose y^* , which player 1 accepts.
 - ▶ Player 2's payoff then equals y^* with one period of delay.
 - ▶ Thus, player 2 would accept any proposal greater or equal to $\delta_2 y_2^*$.

- It follows that

$$x_2^* = \delta_2 y_2^*. \quad (5)$$

- Similarly for player 1:

$$y_1^* = \delta_1 x_1^*. \quad (6)$$

How do we find the equilibrium?

- And since we are bargaining over a pie of size 1, it must be the case that $x_1^* = 1 - x_2^*$.

- We can rewrite (6):

$$1 - y_2^* = \delta_1 x_1^* \quad (7)$$

and from (5) we get:

$$1 - \frac{x_2^*}{\delta_2} = \delta_1 x_1^* \quad (8)$$

- Substituting for x_2^* we get:

$$1 - \frac{1 - x_1^*}{\delta_2} = \delta_1 x_1^* \quad (9)$$

How do we find the equilibrium?

- Simplifying:

$$\delta_2 - (1 - x_1^*) = \delta_1 \delta_2 x_1^* \quad (10)$$

and then:

$$x_1^* - \delta_1 \delta_2 x_1^* = 1 - \delta_2 \quad (11)$$

- Dividing both sides by $1 - \delta_1 \delta_2$ we get:

$$x_1^* = \frac{1 - \delta_2}{1 - \delta_1 \delta_2} \quad (12)$$

Player 1's Strategy

Hence player 1 proposes:

$$\left(\frac{1 - \delta_2}{1 - \delta_1 \delta_2}, \delta_2 \frac{1 - \delta_2}{1 - \delta_1 \delta_2} \right) \quad (13)$$

And accepts player 2's offer if and only if

$$y_1 \geq \delta_1 \frac{1 - \delta_2}{1 - \delta_1 \delta_2} \quad (14)$$

Player 2's Strategy

To get player 2's strategy we then simply use (6) we can see that:

$$y_1^* = \delta_1 \frac{1 - \delta_2}{1 - \delta_1 \delta_2} \quad (15)$$

Hence player 2 proposes:

$$\left(\delta_1 \frac{1 - \delta_2}{1 - \delta_1 \delta_2}, \frac{1 - \delta_1}{1 - \delta_1 \delta_2} \right) \quad (16)$$

And accepts player 1's offer if and only if

$$x_2 \geq \delta_2 \frac{1 - \delta_1}{1 - \delta_1 \delta_2} \quad (17)$$

Baron and Ferejohn: Majority Rule Bargaining, Closed Rule

- **Players:** $N(\text{odd})\text{players}$
- **Terminal Histories:** The proposer in each period makes an offer (x_1, x_2, \dots, x_N)
- **Player Function:**
 - ▶ Random recognition rule: each round player chosen to make proposal with probability $1/N$
 - ▶ Any proposal requires $n = (N + 1)/2$ votes
 - ▶ Feasibility requires $\sum x_i \leq 1$
- **Preferences:** $u_i = \delta x_i^t$

Analyze only Stationary Equilibria in this Game

- A proposer proposes the same division every time she is recognized regardless of the history
- Voters vote only on the basis of the current proposal and expectations about future proposals. Because of assumption 1, future proposals have the same distribution of outcomes in each period.
- Hence the game essentially starts over in every period.

Voter's Continuation Value

- Let v_i be the continuation value for voter i
- Voters do not choose weakly dominated strategies
- Any voter that gets $x_i \geq \delta v$ votes in favour of the proposal
- Any voter that gets $x_i < \delta v$ votes against the proposal

Optimal Proposal

- Proposer gives δv to $n - 1$ other players
- Gives $z = 1 - (n - 1)\delta v$ to the proposer
- And gives 0 to the rest

Compute the continuation value v

- Because the continuation value is just the expected value of the game starting next period, it is simply
 - ▶ z times the probability of being chosen as proposer ($1/N$),
 - ▶ δv times the probability of being included in the winning coalition $(n-1)/N$
 - ▶ And 0 times the remaining probability

Compute the continuation value v

Thus

$$v = \frac{z}{N} + \frac{n-1}{N} \delta v \quad (18)$$

Which yields

$$v = \frac{1}{N} \quad (19)$$

The proposer's share z

And the proposer's share

$$z = 1 - \delta \frac{n-1}{N} = 1 - \delta \frac{N-1}{2N} \quad (20)$$

- One measure of the proposal power is the difference between z and δv or $1 - \delta \frac{(N+1)}{2N}$
- Proposal power increases in N
- Proposal power decreases in δ